• not mentioned in lectures, there is an additional level of complexity introduced by functions that span degenerate irreducible representations
  o the first thing to note is that the components of a degenerate IR are orthogonal
  o for example consider $p_x$ and $p_y$ as basis functions of IR $E$ \[ \int p_x p_y \, d\tau = 0 \]
  o and since IR are orthogonal to each other, the components of a degenerate representation are orthogonal to all the other IR
  o for example if $f_i^E = p_x$ and $f_j^E = p_y$ and $f_k^A = p_z$ then

\[
\begin{align*}
\int f_i^E f_2^E \, d\tau & \propto \delta_{EE} \delta_{i2} = 0 \\
\int f_i^E f_i^A \, d\tau & \propto \delta_{EA} \delta_{i1} = 0 \\
\int f_i^E f_1^E \, d\tau & \propto \delta_{EE} \delta_{i1} \neq 0
\end{align*}
\]

\[
I = \int f_i^E f_j^E \, d\tau \propto \delta_{ij} \delta_{12}
\]

an integral $I = \int f_i^E f_j^E \, d\tau$ over a symmetric range is necessarily zero unless the product $f_i^E f_j^E$ is a basis for the totally symmetric irreducible representation, and this only occurs if $f_i^E = f_j^E$